Factoring by Grouping

Your Name ______________________________
Class ________________ Date ___________

Team Name ______________________________
Team Members:
A) Name:_______________________ B) Name: __________________________
C) Name: _______________________

1. Terms of an algebraic expression are separated by addition, which is indicated by a '+' or '-' sign. The expression $6x^2 + 3x - 12$ has 3 terms.

   How many terms are there in each of the following expressions:
   a) $6x^2y^4$ Answer__________________________
   b) $6x^2 + xy + y^4$ Answer __________________________
   c) $4x^3y^7 - 13x^3y^7 + x^3y^7$ Answer__________________________
   d) $4x^3y^7 + 9x^2y - x - y + xy$ Answer__________________________

2. Each term can have multiple factors (including a numerical coefficient). In $4x^3y^7z + 9x^2yz - yz$, the factors of the first term are 4, x^3, y^7, z

   For $4x^2y^3z + 6x^2yz$, what are the factors of the:
   a) 1st term Answer__________________________
   b) 2nd term Answer__________________________

3. The Distributive Property can be used to factor out common factors from each term of an expression. For example: $ax^2 + bx = x(ax + b)$

   Factor:
   $4x^3 + 12x^2y - 16xy^2$ Answer__________________________

4. The Greatest Common Factor (GCF) is the largest common factor that can be taken out of each term. To find the GCF:

   $\underline{24x^4y^3z - 48x^3y^2z + 36x^2y}$

   - prime factor each term, express the factors using exponents

     $24x^4y^3z = 2^3 3x^4y^3z$
     $48x^3y^2z = 2^4 3x^3y^2z$
     $36x^2y = \underline{2^2 3^2 x^2 y}$

   - for each factor/base that appears in all the terms of the expression use the smallest power of the base

     $2^2 3x^2y$ (z is not included because there aren't any z's in the last term.)

   - simplify the factors.

     GCF = $12x^2y$

What is the GCF of: $27x^4y^3z + 18x^3y^2z^6 - 36x^2y^3z^5 + 54x^3y^2z^3$

Answer___________________________

5. The first step in factoring any expression is to factor out the GCF, if possible.

   $24x^4y^3z - 48x^3y^2z + 36x^2y$ can be factored to: $12\,x^2y(2x^2y^2z - 4xyz + 3)$

   Factor: $27x^4y^3z + 18x^3y^2z^6 - 36x^2y^3z^5 + 54x^3y^2z^3$ Answer___________________________

6. A <u>common factor</u> can also be a <u>binomial</u>. We can factor out the binomial in the same way we would factor out a single variable.

   Single variable: $ax^2 + 7a = a\,(x^2 + 7)$

   Binomial: $x^2\,(x + 2) + 7\,(x + 2) = (x + 2)\,(x^2 + 7)$

   Factor:

   a) $4x^3(y - 2) - 3\,(y - 2)$ Answer___________________________

   b) $6w(z - 5) + 3x(z - 5) - y(z - 5)$ Answer___________________________

7. When the GCF has already been factored out or there are no common factors for <u>all</u> the terms of an expression, additional factoring can be done by <u>GROUPING</u> the terms of the expression and factoring common terms from each group. This is called <u>Factoring by Grouping</u>.

   Factor by Grouping: $\underline{ax + ay + bx + by}$

   group: $[ax + ay] + [bx + by]$

   factor each group: $a(x + y) + b\,(x + y)$

   Factor out the binomial: $(x + y)(a + b)$

Factor by Grouping:

a) $6x^2 + 12xy + 3xy + 6y^2$ Answer______________________________

b) $4x^2 - 6x - 8x + 12$ Answer______________________________

c) $6y^2 + 8y - 3y - 4$ Answer______________________________

8. Factoring by Grouping can be used to factor trinomials using the key number method.

For factorable trinomials of the form $ax^2 + bx + c$

Example A: $10x^2 + 13x + 4$

- The key number is found by multiplying *a* and *c*. $10 \cdot 4 = 40$

- Factor *ac* so that the sum of the two key-number factors = *b*.

ac = 40	b = 13
(8) • (5)	8 + 5

- Replace the middle term with two terms whose numerical coefficients are the sum of the numerical coefficient of the original middle term. $10x^2 + 8x + 5x + 4$

- Factor by grouping. $2x(5x + 4) + (5x + 4)$ [same as $1(5x + 4)$]
$= (5x + 4)(2x + 1)$

Factor: $3x^2 + 14x + 8$ Answer_______________________

Example B: In this problem *c* is negative.

- Find the key number by multiplying *a* and *c*.

$$\underline{10x^2 - 3x - 4}$$
$$10 \bullet (-4) = -40$$

- Factor *ac* so that the sum of the two key-number factors = *b*.
  -Since *ac* = a negative number, one factor will be negative and one will be positive.
  -Since *b* = a negative number, the larger factor will be negative.

ac = - 40	b = -3
(-8) • (5)	-8 + 5

- Replace the middle term with two terms whose numerical coefficients are the sum of the numerical coefficient of the original middle term.

$$10x^2 - 8x + 5x - 4$$

- Factor by grouping.

$$2x(5x - 4) + (5x - 4)$$
$$(5x - 4)(2x + 1)$$

Factor: $6x^2 + 5x - 6$ Answer_______________________

Example C: Has a large key number: $\underline{14x^2 + 27x - 20}$

- Find the key number by multiplying *a* and *c*.

$14 \bullet (-20) = -280$

- Factor *ac* so that the sum of the two key-number factors = *b*.
  - Since *ac* = a negative number, one factor will be negative and one will be positive.
  - Since *b* = a positive number, the larger factor will be positive.
  - To help determine the key number, prime factor 280
  - Group the factors so they add to 27

ac = -280	b = 27

Not easy to see, so use prime factoring

$280 = 2^3 \cdot 5 \cdot 7$

$2^3 = 8, \quad 5 \cdot 7 = 35$

$35 + (-8) = 27$

ac = -280	b = 27
(35)(-8)	35 + (-8)

- Replace the middle term with two terms whose numerical coefficients are the sum of the numerical coefficient of the original middle term.

$14x^2 + 35x - 8x - 20$

- Factor by grouping.

$7x(2x + 5) - 4(2x + 5)$ [factor out - 4 in order to have (2x + 5) in the 2nd term]

$= (2x + 5)(7x - 4)$

Factor: $10x^2 - 3x - 18$

Answer____________________________

Let's put it all together, remember the first step is to find the Greatest Common Factor.

Factor: $18x^2y + 15xy - 18y$ Answer__________________________