Practice Exam #5 On Chapters 9 and 10

1. Find the indicated root, or state that the expression is not a real number. $\sqrt{169}$
   SOLUTION! $\sqrt{169} = \sqrt{13^2} = 13$

2. Find the indicated root, or state that the expression is not a real number. $\sqrt{-81}$

   SOLUTION! $\sqrt{-81}$ is not a real number because there is no real number squared that is equal to -81. The square root of a negative number is not a real number.

3. Reduce: $\sqrt{63y^5}$

   SOLUTION! $\sqrt{63y^5} = \sqrt{3^2 \cdot 7^1 \cdot y^4 \cdot y^1} = 3y^2\sqrt{7y}$

4. Reduce: $\sqrt{169a^{25}b^{64}}$

   SOLUTION! $\sqrt{169a^{25}b^{64}} = \sqrt{13^2 \cdot a^{24} \cdot a^1 \cdot b^{64}} = 13a^{12}b^{32}\sqrt{a}$

5. Simplify: $\sqrt{48}$

   SOLUTION! $\sqrt{48} = \sqrt{2^4 \cdot 3^1} = 2^2\sqrt{3} = 4\sqrt{3}$

6. Simplify: $\sqrt{24} + 3\sqrt{54}$

   SOLUTION! $\sqrt{24} + 3\sqrt{54}$
   $= \sqrt{2^3 \cdot 3^1} + 3\sqrt{2^1 \cdot 3^3}$
   $= \sqrt{2^2 \cdot 2^1 \cdot 3^1} + 3\sqrt{2^1 \cdot 3^2 \cdot 3^1}$
   $= 2\sqrt{6} + 3 \cdot 3\sqrt{6}$
   $= 2\sqrt{6} + 9\sqrt{6}$
   $= 11\sqrt{6}$

7. Simplify: $11\sqrt{20} + 12\sqrt{20} - 25\sqrt{20}$

   SOLUTION! $11\sqrt{20} + 12\sqrt{20} - 25\sqrt{20}$
   $= -2\sqrt{20} = -2\sqrt{2^2 \cdot 5^1} = -2 \cdot 2\sqrt{5} = -4\sqrt{5}$

8. Simplify: $-13y\sqrt{54y^3} + 10\sqrt{24y^5} - 8y^2\sqrt{216y}$

   SOLUTION!
   $-13y\sqrt{54y^3} + 10\sqrt{24y^5} - 8y^2\sqrt{216y}$
   $= -13y\sqrt{3^2 \cdot 3^1 \cdot 2^1 \cdot y^2 \cdot y^1} + 10\sqrt{2^2 \cdot 2^1 \cdot 3^1 \cdot y^4 \cdot y^1} - 8y^2\sqrt{2^2 \cdot 2^1 \cdot 3^2 \cdot 3^1 \cdot y}$
   $= -13y \cdot 3 \cdot y \cdot \sqrt{6y} + 10 \cdot 2 \cdot y^2\sqrt{6y} - 8y^2 \cdot 2 \cdot 3\sqrt{6y}$
   $= -39y^2\sqrt{6y} + 20y^2\sqrt{6y} - 48y^2\sqrt{6y}$
   $= -67y^2\sqrt{6y}$

9. Multiply and then simplify: $\sqrt{8}\,(8\sqrt{2} - 6\sqrt{6})$

SOLUTION! $\sqrt{8}\cdot(8\sqrt{2} - 6\sqrt{6})$

$= 8\sqrt{2}\cdot\sqrt{8} - 6\sqrt{6}\cdot\sqrt{8}$

$= 8\sqrt{16} - 6\sqrt{48}$

$= 8\cdot 4 - 6\sqrt{2^4\cdot 3^1}$

$= 32 - 6\cdot 2^2\sqrt{3}$

$= 32 - 24\sqrt{3}$

10. Multiply and then simplify: $(7 - \sqrt{5})(10 + 3\sqrt{5})$

SOLUTION! $(7 - \sqrt{5})(10 + 3\sqrt{5})$

$= 7\cdot 10 + 7\cdot 3\sqrt{5} - 10\sqrt{5} - \sqrt{5}\cdot 3\sqrt{5}$

$= 70 + 21\sqrt{5} - 10\sqrt{5} - 3\cdot 5$

$= 55 + 11\sqrt{5}$

11. Multiply and then simplify: $(\sqrt{5x} + 3\sqrt{8})\cdot(\sqrt{5x} - 3\sqrt{2})$

SOLUTION! $(\sqrt{5x} + 3\sqrt{8})\cdot(\sqrt{5x} - 3\sqrt{2})$

$= 5x - 3\sqrt{5x\cdot 2} + 3\sqrt{8\cdot 5x} - 3\cdot 3\sqrt{8\cdot 2}$

$= 5x - 3\sqrt{10x} + 3\sqrt{40x} - 9\cdot\sqrt{16}$

$= 5x - 3\sqrt{10x} + 3\sqrt{4\cdot 10x} - 9\cdot 4$

$= 5x - 3\sqrt{10x} + 6\sqrt{10x} - 36$

$= 5x + 3\sqrt{10x} - 36$

12. Divide and then simplify: $\dfrac{\sqrt{80x^4}}{\sqrt{2x^2}}$

SOLUTION! $\dfrac{\sqrt{80x^4}}{\sqrt{2x^2}} = \sqrt{\dfrac{80x^4}{2x^2}} = \sqrt{40x^2} = \sqrt{2^2\cdot 2^1\cdot 5^1\cdot x^2} = 2\cdot x\sqrt{2\cdot 5} = 2x\sqrt{10}$

13. Rationalize the denominator: $\dfrac{10}{\sqrt{5}}$

SOLUTION! $\dfrac{10}{\sqrt{5}} = \dfrac{10}{\sqrt{5}}\cdot\dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{10\sqrt{5}}{5} = 2\sqrt{5}$

14. Rationalize the denominator: $\dfrac{10}{\sqrt{50}}$

SOLUTION!

$$\frac{10}{\sqrt{50}} = \frac{10}{\sqrt{2\cdot 5^2}} = \frac{10}{5\sqrt{2}} = \frac{10}{5\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{5\cdot 2} = \frac{10\sqrt{2}}{10} = \sqrt{2}$$

15. Rationalize the denominator: $\frac{124}{8-\sqrt{2}}$

SOLUTION!

$$\frac{124}{8-\sqrt{2}} = \frac{124}{8-\sqrt{2}} \cdot \frac{8+\sqrt{2}}{8+\sqrt{2}} = \frac{124(8+\sqrt{2})}{64-2} = \frac{124(8+\sqrt{2})}{62} = 2(8+\sqrt{2})$$

$= 16 + 2\sqrt{2}$

16. $\sqrt{x+2} = -6$

SOLUTION!

$\sqrt{x+2} = -6$
$(\sqrt{x+2})^2 = (-6)^2$
$x + 2 = 36$
$x = 36 - 2$
$x = 34$

MUST CHECK 34
$\sqrt{x+2} = -6$
$\sqrt{34+2} \stackrel{?}{=} -6$
$\sqrt{36} \neq -6$

Because 34 is an extraneous solution and is thus invalid, this equation has NO SOLUTION.

17. Solve for x and check your solution: $\sqrt{-2x-2} = x+5$

SOLUTION!

$\sqrt{-2x-2} = x+5$
$(\sqrt{-2x-2})^2 = (x+5)^2$
$-2x - 2 = (x+5)(x+5)$
$-2x - 2 = x^2 + 10x + 25$
$0 = x^2 + 10x + 2x + 25 + 2$
$0 = x^2 + 12x + 27$
$0 = (x+3)(x+9)$
either $x + 3 = 0$ *or* $x + 9 = 0$
$x = -3$ or $x = -9$
The only valid solution is $x = -3$.

MUST CHECK −3
$\sqrt{-2x-2} = x+5$
$\sqrt{-2(-3)-2} \stackrel{?}{=} -3+5$
$\sqrt{4} = 2$✓

MUST CHECK −9
$\sqrt{-2x-2} = x+5$
$\sqrt{-2(-9)-2} \stackrel{?}{=} -9+5$
$\sqrt{16} \neq -4$

18. Solve for x and check your solution: $5\sqrt{x+5} = 10\sqrt{2x-18}$

SOLUTION!

$5\sqrt{x+5} = 10\sqrt{2x-18}$
$(5\sqrt{x+5})^2 = (10\sqrt{2x-18})^2$
$25(x+5) = 100(2x-18)$
$25x + 125 = 200x - 1,800$
$25x - 200x = -1,800 - 125$
$-175x = -1,925$
$\frac{-175x}{-175} = \frac{-1,925}{-175}$
$x = 11$ is a valid solution.

MUST CHECK 11
$5\sqrt{x+5} = 10\sqrt{2x-18}$
$5\sqrt{11+5} \stackrel{?}{=} 10\sqrt{22-18}$
$5\sqrt{16} \stackrel{?}{=} 10\sqrt{4}$
$5 \cdot 4 \stackrel{?}{=} 10 \cdot 2$
$20 = 20$✓

19. The time, t, in seconds for a free-falling object to fall d feet is modeled by the formula $t = \sqrt{\frac{d}{16}}$. If a worker accidentally drops a hammer from a building and it hits the ground after 4 seconds, from what height was the hammer dropped?

SOLUTION!

$$t = \sqrt{\frac{d}{16}}$$

$$4 = \sqrt{\frac{d}{16}}$$

$$(4)^2 = \left(\sqrt{\frac{d}{16}}\right)^2$$

$$16 = \frac{d}{16}$$

$$\frac{16}{1} = \frac{d}{16}$$

$$d = 256$$

The hammer was dropped from 256 feet. I hope no one was underneath!

20. Solve for x by taking the square root on both sides. $x^2 = 81$

SOLUTION!

$x^2 = 81$
$\sqrt{x^2} = \sqrt{81}$
$|x| = 9$
$x = 9$ or $x = -9$
$x = \pm 9$

21. Solve for x by taking the square root on both sides. $(x+3)^2 = 49$

SOLUTION!

$(x+3)^2 = 49$
$\sqrt{(x+3)^2} = \sqrt{49}$
$|x+3| = 7$
$x + 3 = 7$ or $x + 3 = -7$
$x = 7 - 3$ or $x = -7 - 3$
$x = 4$ or $x = -10$

22. Solve for x by taking the square root on both sides. $(x-7)^2 = 40$

SOLUTION!

$(x-7)^2 = 40$
$\sqrt{(x-7)^2} = \sqrt{40}$
$|x-7| = 2\sqrt{10}$
$x-7 = 2\sqrt{10}$ or $x-7 = -2\sqrt{10}$
$x = 7+2\sqrt{10}$ or $x = 7-2\sqrt{10}$
$x = 7 \pm 2\sqrt{10}$

23. What is the width of a 15-inch television set whose height is 9 inches?

SOLUTION!

Let $x = \textit{the width}$
$x^2 + 9^2 = 15^2$
$x^2 + 81 = 225$
$x^2 = 144$
$\sqrt{x^2} = \sqrt{144}$
$|x| = 12$
$x = 12\, or\, x = -12\,(reject!)$
$The\, width\, of\, the\, television\, set\, is\, 12\, inches.$

24. Solve for x by completing the square: $x^2 + 10x + 21 = 0$

SOLUTION!

$x^2 + 10x + 21 = 0$
$x^2 + 10x + \square = -21 + \square$
$x^2 + 10x + \boxed{25} = -21 + \boxed{25}$ $b = 10 \ldots\ c = \left(\frac{1}{2}b\right)^2 = \left(\frac{1}{2}\cdot 10\right)^2 = 5^2 = 25$
$(x+5)^2 = 4$
$\sqrt{(x+5)^2} = \sqrt{4}$
$|x+5| = 2$
$x+5 = 2$ or $x+5 = -2$
$x = -5+2$ or $x = -5-2$
$x = -3$ or $x = -7$

25. Solve for x by completing the square: $x^2 + 5x - 2 = 0$

SOLUTION!

$x^2 + 5x - 2 = 0$
$x^2 + 5x + \square = 2 + \square$

$x^2 + 5x + \boxed{\frac{25}{4}} = 2 + \boxed{\frac{25}{4}}$ $b = 5 \ldots\ c = \left(\frac{1}{2}b\right)^2 = \left(\frac{1}{2}\cdot 5\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$

$4\cdot x^2 + 4\cdot 5x + 4\cdot \frac{25}{4} = 4\cdot 2 + 4\cdot \frac{25}{4}$
$4x^2 + 20x + 25 = 8 + 25$

$(2x+5)(2x+5) = 33$
$(2x+5)^2 = 33$
$\sqrt{(2x+5)^2} = \sqrt{33}$
$|2x+5| = \sqrt{33}$
$2x+5 = \sqrt{33}$ or $2x+5 = -\sqrt{33}$
$2x = -5+\sqrt{33}$ or $2x = -5-\sqrt{33}$

$$x = \frac{-5+\sqrt{33}}{2} \text{ or } x = \frac{-5-\sqrt{33}}{2}$$

$$x = \frac{-5 \pm \sqrt{33}}{2}$$

26. Solve by applying the quadratic formula: $2x^2 + 9x - 5 = 0$

SOLUTION!

$2x^2 + 9x - 5 = 0$ $\quad a = 2,\ b = 9,\ c = -5$

$$x = \frac{-b \pm \sqrt{b^2 + (-4)ac}}{2a} \quad x = \frac{-9 \pm \sqrt{9^2 + (-4)\cdot 2 \cdot (-5)}}{2\cdot 2} \rightarrow$$

$$x = \frac{-9 \pm \sqrt{81+40}}{4} \quad x = \frac{-9 \pm \sqrt{121}}{4} \quad x = \frac{-9 \pm 11}{4} \rightarrow$$

$$x = \frac{-9+11}{4} \ or\ x = \frac{-9-11}{4}$$

$$x = \frac{2}{4} \ or\ x = \frac{-20}{4}$$

$$x = \frac{1}{2} \ or\ x = -5$$

27. Solve by applying the quadratic formula: $3x^2 - 10x + 2 = 0$

SOLUTION!

$3x^2 - 10x + 2 = 0$ $\quad a = 3,\ b = -10$ or $c = 2$ $\rightarrow$

$$x = \frac{-b \pm \sqrt{b^2 + (-4)ac}}{2a} \quad x = \frac{-(-10) \pm \sqrt{(-10)^2 + (-4)\cdot 3 \cdot 2}}{2\cdot 3} \rightarrow$$

$$x = \frac{10 \pm \sqrt{100 + (-24)}}{6} \quad x = \frac{10 \pm \sqrt{76}}{6} \quad x = \frac{10 \pm \sqrt{2^2 \cdot 19}}{6} \rightarrow$$

$$x = \frac{10 \pm 2\sqrt{19}}{6} \quad x = \frac{2\cdot(5 \pm \sqrt{19})}{2\cdot 3} \quad x = \frac{(5 \pm \sqrt{19})}{3}$$

28. A rectangular bedroom has a length which measures three feet more than its width. If the area of the rectangle is 180 square feet, find its dimensions.

SOLUTION!

Let $x =$ the width
Then $x + 3 =$ the length
$x(x+3) = 180$
$x^2 + 3x - 180 = 0$
$(x+15)(x-12) = 0$
either $x + 15 = 0$ or $x - 12 = 0$
$x = -15$ or $x = 12$
$x = -15$ is an impossible solution and is discarded.

The width is 12 feet and the length is 15 feet.

Bonus: Solve for x by completing the square: $3x^2 - 5x - 2 = 0$

SOLUTION!

$3x^2 - 5x - 2 = 0$

$$\frac{3x^2}{3} - \frac{5x}{3} - \frac{2}{3} = \frac{0}{3}$$

$$x^2 - \frac{5}{3}x - \frac{2}{3} = 0$$

The quadratic term must have a coefficient of 1!

$$x^2 - \frac{5}{3}x + \square = \frac{2}{3} + \square$$

$$x^2 - \frac{5}{3}x + \boxed{\tfrac{25}{36}} = \frac{2}{3} + \boxed{\tfrac{25}{36}}$$

$$b = -\frac{5}{3} \ldots \ c = \left(\frac{1}{2} \cdot b\right)^2 \ c = \left(\frac{1}{2} \cdot \frac{-5}{3}\right)^2 = \left(\frac{-5}{6}\right)^2 = \frac{25}{36}$$

$$36 \cdot x^2 + 36 \cdot \left(-\frac{5}{3}x\right) + 36 \cdot \frac{25}{36} = 36 \cdot \frac{2}{3} + 36 \cdot \frac{25}{36}$$

$$36x^2 - 60x + 25 = 24 + 25$$

$(6x-5)(6x-5) = 49$
$(6x-5)^2 = 49$
$\sqrt{(6x-5)^2} = \sqrt{49}$
$|6x - 5| = 7$
$6x - 5 = 7$ or $6x - 5 = -7$

$$x = \frac{5+7}{6} \text{ or } x = \frac{5-7}{6}$$

$$x = 2 \text{ or } x = -\frac{1}{3}$$